

# Quantitative Analysis of Heart Sounds and Systolic Heart Murmurs Using Wavelet Transform and AR Modeling

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**Abstract**—A quantitative approach integrating AR modeling and wavelet transform is presented in this paper to analyze the digitized phonocardiogram. The recognition of the first and the second heart sounds ( $S_1$  and  $S_2$ ) were facilitated with wavelet transform without referring to the QRS waveform. We found that the Daubechies wavelet is most effective in identifying  $S_1$  and  $S_2$ . In addition, the boundaries of  $S_1$ ,  $S_2$ , and the onset and duration of the systolic murmur thus identified within the systole could be marked using the wavelet-filtered signal's strength. Furthermore, quantitative measures derived from a 2<sup>nd</sup> order AR model were used to delineate the configuration and pitch of the systolic murmur found within through piecewise segmentation. The proposed approach was tested and proved effective in delineating a set of clinically diagnosed systolic murmurs. The suggested AR and wavelet transform combined approach can be generalized with minor adjustments to delineate diastolic murmurs as well.

## I. INTRODUCTION

MODERN electronic technologies have emerged to assist the diagnosis of heart diseases, e.g., the electrocardiogram (ECG), magnetic resonance imaging (MRI), and the echocardiogram, to name a few. However, there is still important auditory information that physicians utilize to understand the cardiovascular system's condition. Heart sounds, heart murmurs, and turbulence in the blood flow are detected in this way. In addition, the cost and scarce availability of expensive diagnostic tools render the traditional stethoscope-based cardiac auscultation a vital investigative process for physicians.

To date, cardiac auscultation is still the most commonly utilized bedside investigative process by physicians to reveal crucial information regarding the well-being of the cardiovascular system and to detect signs of cardiovascular defects or alternations [1], [2]. However, auscultation based on human auditory sensitivity is heavily influenced by an individual's subjective judgment, rather than objective assessment, because it is susceptible to each physician's auditory sensitivity, clinical experience, and quality of equipment. Key features of heart sounds and murmurs, such as timing, duration, intensity, pitch, and configuration [1], [2] are to be determined by listening to cardiac cycles that each

last for less than a second.

Possible improvements include the use of modern digital signal processing algorithms to extract quantitative measures that closely relate to the diagnostic features used in cardiac auscultation. For example, the use of wavelet transforms (WT) in the field of examining heart sounds has been done by many researchers [5], [6], [9], [12], [15].

A heartbeat is the combination of the heart contracting (systole) and relaxing (diastole); these two periods are marked by the first and the second heart sounds,  $S_1$  and  $S_2$ . Systole indicates the period that begins after  $S_1$  and ends before  $S_2$ ; diastole takes place after  $S_2$  and ends before  $S_1$ . Systolic murmurs occur during systole and are the focus of this paper. To detect the onset and duration of a systolic murmur, one needs to determine where the systole interval is located in the cardiac cycle of a digitized phonocardiogram. Clinical observations show that  $S_1$  is louder than  $S_2$  at the apex of the heart whereas  $S_2$  is louder than  $S_1$  at the base [2], also the length of systole is usually shorter than the length of diastole; the pitch (frequency) of  $S_1$  and  $S_2$  is generally lower than the pitch of the murmur [2], [7], [8].

Based on the aforementioned understanding, an effective way to facilitate the identification of heart sounds and systolic murmurs is applying the wavelet transforms [10]–[13] with an appropriate mother wavelet and the corresponding scaling depth [13]. In this paper, we used the Daubechies wavelet (*db6*) at the fourth level of decomposition. We also used autoregressive modeling to delineate the pitch profile of systolic murmurs by piecewise segmentation. The Burg method was implemented to estimate the 2<sup>nd</sup> order AR model coefficient to derive the most dominant frequency of each short segment [3]. The effectiveness of the combined AR and WT signal processing approach was tested with different systolic murmurs. Demonstrated results on a ventricular septal defect (VSD) and an early systolic murmur are presented.

## II. METHODS

### A. Wavelet Transform

Wavelet transform has gained much popularity in recent years in a wide spectrum of applications. Researchers from different disciplines such as mathematics and signal processing have contributed to the development of many useful wavelet methods for multi-scale signal analysis and joint time-frequency representation of signals [9]–[14]. Whereas Fourier transform treats each signal of interest as a linear combination of sinusoids having different frequencies weighted by different amplitudes, WT approximates the

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signal of interest as a combination of scaled and delayed versions of a chosen mother wavelet. Per application, different types of mother wavelets can be adopted for that purpose. For a given signal  $x(t)$ , the continuous-time wavelet transform (CWT) is described below

$$CWT_x(\tau, a) = \frac{1}{\sqrt{a}} \int x(t) h^*\left(\frac{t-\tau}{a}\right) dt, \quad (1)$$

where  $h^*((t-\tau)/a)$  is the complex conjugate of the scaled and shifted version of a chosen mother wavelet appropriate for the signal  $x(t)$  [11], [13]. Wavelet analysis provides some additional signal analysis advantages not shared by Fourier analysis, e.g., it can localize the information of interest in the joint time-frequency domain; in particular, it can trade time-resolution for frequency-resolution when needed. Such flexibility makes it an excellent tool of choice to analyze non-stationary signals and to detect signal transition. Heart sound and murmur signals have frequency contents that change over time and therefore a joint time-frequency wavelet transform (WT) is a good match for heart sound analysis [14]–[17].

The wavelet transform uses short data windows for high frequency signals and long data windows for low frequency components. This unique performance is useful in heart sound analysis to facilitate the separation of a typical cardiac cycle into heart sounds  $S_1$  and  $S_2$  and murmurs, which have higher pitch frequencies than  $S_1$  and  $S_2$ . WT based heart sound and murmur analysis methods have been emerging recently in analyzing cardiovascular signals having physiological alternations such as turbulent sounds caused by femoral artery stenoses [6], WT-based spectrogram to examine heart signals having wavelet scale variation in time (i.e., scaleograms) [14], decomposition and segmentation of heart murmurs [15], and boundary identification of  $S_1$  and  $S_2$  [16].

The wavelet transform was used here to preprocess the digitized phonocardiogram. Following the multi-resolution signal decomposition idea proposed by Mallat [13], which uses dyadic (based on powers of two) scales and positions to make the analysis efficient without loss of accuracy, we tested different mother wavelets and found that the Daubechies wavelet is most effective in identifying  $S_1$  and  $S_2$ . It works based on the fact that typical heart sounds  $S_1$  and  $S_2$  usually reside in the frequency range below 60-80 Hertz, whereas heart murmurs are mostly higher than 125 Hertz. For a signal waveform that encompasses the whole cardiac cycle, a lower resolution signal can be derived by low-pass filtering with a half-band low-pass filter, where two outputs (details and approximations) are separated into higher and lower frequency components, respectively.

One can accurately locate the boundaries of heart sounds  $S_1$  and  $S_2$  by checking the filtered signal's strength. For each short data segment, an energy index to characterize the amplitude variation is given in (2):

$$E_x = \frac{1}{N} \sum_{k=1}^N (x(k) - \mu_x)^2, \quad (2)$$

where  $\mu_x$  is the mean value of the data segment. The energy index coincides with the statistical definition of variance.

Even with proven evidence, the effectiveness of WT filtering, however, hinges upon two key parameters: the choice of the mother wavelet and the scale depth applied. Each mother wavelet has its own center frequency. As the scale increases, the band of the retained signal decreases; a pseudo-frequency corresponds to each particular scale  $a$ .

We found through our experience that the Daubechies (*db6*) wavelet with the fourth level of decomposition is most effective for isolating  $S_1$  and  $S_2$  from heart murmurs. At the fourth level of decomposition, the corresponding pseudo-frequency is lower than the murmurs but higher than  $S_1$  and  $S_2$  heart sounds [16]. Murmurs with high-frequency contents are greatly attenuated from the WT-filtered approximations, while  $S_1$  and  $S_2$  are not.

To characterize the systolic murmur by its intensity, we used the energy index in (2). The energy index is the mean-squared value of a signal and can be effectively utilized to determine the onset and duration of systole interval and to provide a configuration profile for the systolic murmur found within.

### B. Autoregressive Modeling

Autoregressive (AR) models are widely used in many applications, such as speech analysis, linear predictive coding, power spectral analysis, etc. AR modeling is often the choice of application because of the ease of implementation and the efficiency of computing AR coefficients [3]–[4]. The Burg AR modeling algorithm [3] was implemented in this paper to compute the AR coefficients of the digitized phonocardiogram data,  $\{x(n)\}$ . Burg's method estimates the optimal AR coefficients by way of minimizing the combined forward and backward prediction errors, which are, for the  $p^{\text{th}}$  order AR model, respectively

$$e_f(k) = x(k) - \sum_{i=1}^p a_i^p x(k-i),$$

and

$$e_b(k) = x(k) - \sum_{i=1}^p a_i^p x(k+i), \quad (3)$$

The AR model coefficients  $\{a_i^p\}$  are computed by minimizing the performance index shown in (4) with respect to reflection coefficients  $\{a_i^i\}$ , and the resultant optimal AR coefficients are updated by the Levinson-Durbin recursive formula [3].

$$LPE : J^p = \sum_k (e_f(k)^2 + e_b(k)^2), \quad (4)$$

As mentioned earlier, AR model coefficients can be used for many purposes. The poles of AR models were used to quantify the hippocampal EEG frequency during REM sleep [4] and to detect the sounds associated with turbulent blood flow [5]. Since it is of clinical interest to physicians during cardiac

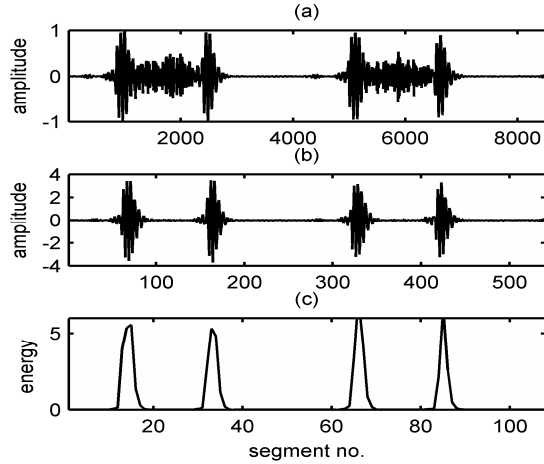


Fig. 1. Ventricular septal defect: (a) heart sounds and murmurs, (b) filtered signal with db6 wavelet, and (c) energy index.

auscultation to describe the pitches and configurations of murmurs, we used in this paper a 2<sup>nd</sup> order AR model. The digitized phonocardiogram was first divided into evenly-sized short segments, and the AR model coefficients were estimated for each short segment using the Burg method. The two AR model coefficients  $\{a_2, a_1\}$  were used to determine a pair of complex conjugate poles and to calculate their corresponding frequencies. The pitch frequency (Hz) was adjusted by the sampling frequency ( $f_{samp}$ ) as follows [4],

$$pitch\_freq = \frac{f_{samp}}{2} * \tan^{-1}\left(\frac{\sqrt{4a_2 - a_1^2}}{a_1}\right) / \pi. \quad (5)$$

In addition to the estimated frequency derived from the angle of the poles, the magnitude of pole (varying between 0 and 1) also serves as a good indicator in regard to the frequency bandwidth. For example, poles having magnitudes near 1 reflect narrow-band energy concentration such as a pure sine wave, whereas smaller pole magnitudes indicate otherwise. The resultant linear prediction error (LPE) described in (4) can be used as a quantitative measure for the output of a 2<sup>nd</sup> order AR whitening filter.

In summary, the steps in our approach are:

- Performing WT-filtering using the *db6* wavelet described above.
- Using the energy index in (2) to isolate  $S_1$  and  $S_2$  and determine the onset and duration of the systole.
- Setting  $S_1$  and  $S_2$  boundaries at about 5% to 7.5% of the peak energy and within 55 milliseconds from the heart sound peak.
- Dividing the complete cardiac cycle into smaller data segments of equal length and performing AR modeling analysis to derive quantitative measures as described in (4) and (5).

### III. RESULTS AND DISCUSSION

We have used the combined WT and AR modeling methods to analyze several systolic murmurs: atrial septal

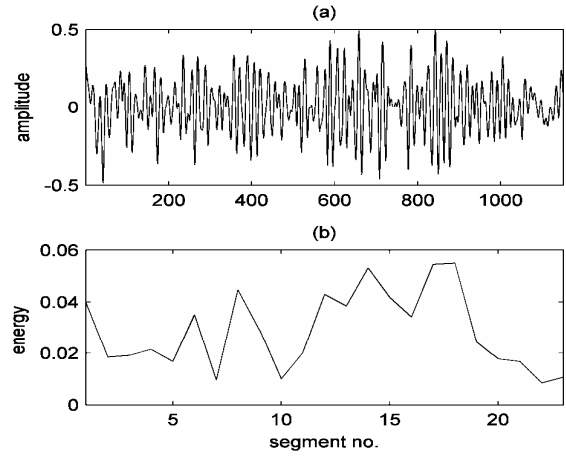


Fig. 2. (a) Systolic murmur of ventricular septal defect, and (b) energy profile by segments.

defect, ventricular septal defect, innocent murmur, early systolic murmur, mid-systolic murmur, and Still's murmur. Analysis results of ventricular septal defect (VSD) and early systolic murmur are shown in Figures 1-5. The heart sounds and murmur signals analyzed in this paper were selected from [2] with a sampling frequency of 4.41 kHz and a 16-bit quantization.

Two cardiac cycles displaying the ventricular septal defect murmur are shown in Fig. 1a. The result of using the *db6* wavelet at the fourth level of decomposition to process the original data is shown in Fig. 1b, which effectively isolates  $S_1$  and  $S_2$ . The energy index in Fig. 1c was used to clearly identify  $S_1$  and  $S_2$  and the onset and duration of the systole. The systolic murmur extracted from Fig. 1a is shown in Fig. 2a, and the energy indices by segments were calculated and are shown in Fig. 2b. Each short segment contains 50 data samples and adjacent segments were not overlapped.

Results obtained from AR modeling are summarized in Fig. 3. The frequency of the murmur is found mostly around 370 Hz (Fig. 3b) with pole magnitudes are between 0.97 and 1 throughout the murmur (Fig. 3a). The high pole magnitudes

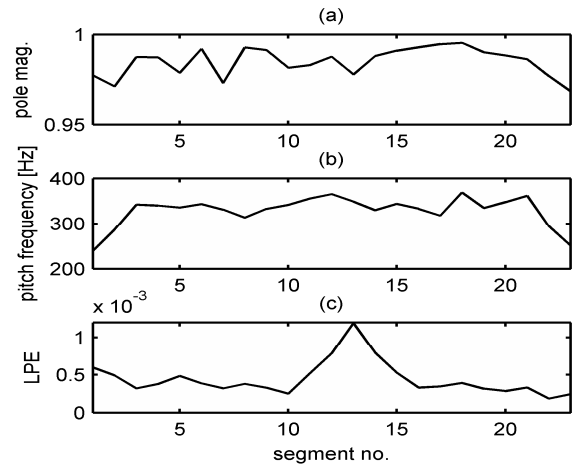


Fig. 3. Quantitative measures generated by 2<sup>nd</sup> order AR models for VSD by segments: (a) magnitude of poles, (b) pitch frequency, and (c) linear prediction errors.

( $> 0.95$ ) suggest that the VSD murmur consists of narrow-band frequency components around 370 Hz. The LPE measures in Fig. 3c show the murmur's intensity being relatively constant with a slight peak in the center, i.e., a crescendo-decrescendo configuration between the tenth and fifteenth segments.

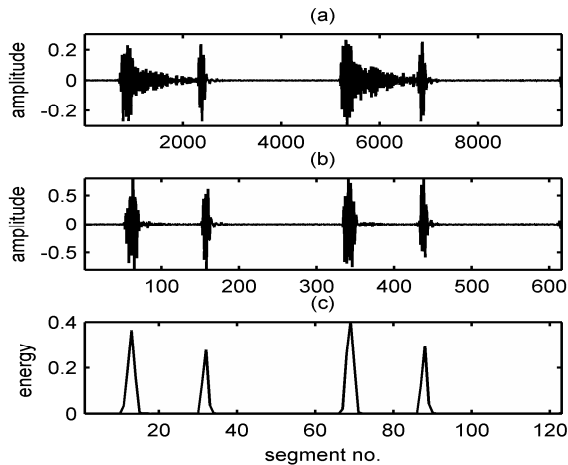


Fig. 4. Early systolic murmur: (a) heart sounds and murmurs, (b) WT filtered signal with db6 wavelet, and (c) energy index.

The same signal processing steps were applied to an early systolic murmur (Fig. 4a) which begins immediately after  $S_1$  and ends prior to  $S_2$ . Its loudness is maximal at the beginning and progressively decreases. After WT-filtering,  $S_1$  and  $S_2$  are better revealed in Fig. 4b. The energy index of the filtered signal clearly marks the occurrence of  $S_1$  and  $S_2$  in Fig. 4c, and the systole and the diastole as well. Quantitative measures derived from 2<sup>nd</sup> order AR models are summarized in Fig. 5. The systolic murmur pitch (Fig. 5a) is above 400 Hz

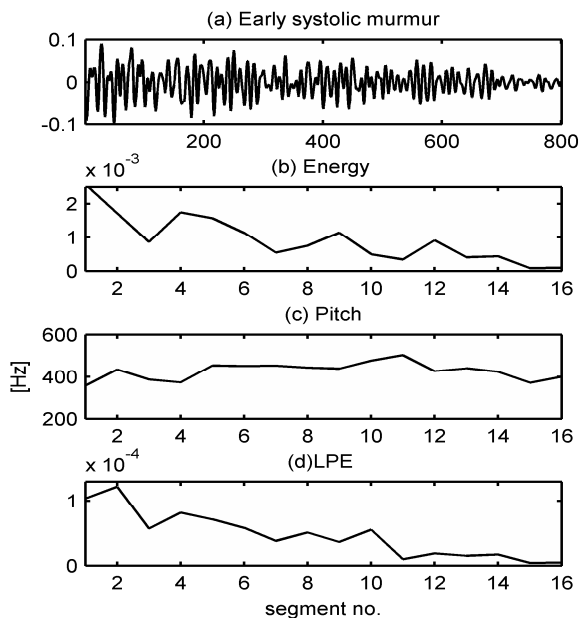


Fig. 5. Quantitative measures by segments generated using 2<sup>nd</sup> order AR models for an early systolic murmur: (a) murmur, (b) energy index, (c) pitch frequency and (d) linear prediction errors. (LPE).

during systole (Fig. 5c). The pole magnitudes are also near one. It is interesting to note that both the energy index and the LPE in Fig. 5b and Fig. 5d accurately delineate the murmur configuration as decrescendo.

#### IV. CONCLUSION

AR modeling and WT were used in this paper to demonstrate the effectiveness of extracting useful diagnostic features of systolic murmurs. With the choice of the Daubechies wavelet, WT is very effective in facilitating the identification of heart sounds, systole, and diastole. AR modeling is computationally efficient to generate useful features for systolic murmurs. Although only systolic murmurs were examined in this paper, the suggested approach can be generalized with a little modification to delineate both systolic and diastolic murmurs.

#### REFERENCES

- [1] A. R. Freeman and S. A. Levin, "The clinical significance of the systolic murmur," *Ann Intern Med.*, 1933:6, 1371-1385.
- [2] D. Mason. *Listening to the Heart: Heart Sounds and Murmurs*, Davis F A, Aug., 2000.
- [3] J. P. Burg, *A new analysis technique for time series data*, in Modern Spectrum Analysis (D. G. Childers, ed.), IEEE Press, New York, 1978, 42-48.
- [4] T. Ning and J. D. Bronzino, "Autoregressive and bispectral analysis techniques: EEG applications," *Special Issue on Biomedical Signal Processing for IEEE Engineering in Medicine and Biology Magazine*, Vol.9, No.1, pp.47-50, March 1990.
- [5] Y. M. Akay and M. Akay et al "Noninvasive acoustical detection of coronary artery disease: a comparative study of signal processing methods," *IEEE Trans. on Biomed. Eng.*, Vol.40, No.6, pp.571-578, June, 1993.
- [6] Y. M. Akay, et al. "Investigating the effects of vasodilator drugs on the turbulent sound caused by femoral artery stenosis using short-term Fourier and wavelet transform methods," *IEEE Trans. on Biomed. Eng.*, Vol.41, No.10, pp.921-928, October, 1994.
- [7] T. Ning and K.S. Hsieh, "Delineation of systolic murmurs by autoregressive modeling," Proc. IEEE 21<sup>st</sup> Northeast Bioeng. Conf., pp.19-21, Bar Harbor, Maine, May 22-23, 1995.
- [8] N. Atanasov and T. Ning, "Quantitative delineation of heart murmurs using features derived from autoregressive modeling," *Proc. IEEE 33th Annual Northeast Bioeng. Conf.*, pp.167-168, Stony Brook, March 10-11, NY, 2007.
- [9] M. Unser and A. Aldroubi, "A review of wavelets in biomedical applications", *Proc. IEEE*, vol. 84, no. 4, April 1996.
- [10] M. Vetterli and J. Kovacevic, *Wavelets and Subband Coding*, Englewood Cliffs, NJ: Prentice Hall, 1995.
- [11] S. Qian and D. Chen, *Joint Time-frequency Analysis: Methods and Applications*, NJ: Prentice Hall, 1996.
- [12] C. W. Lang and K. Forinash, "Time-frequency analysis with the continuous wavelet transform." *Am J Phys* 66(9):794-797, 1998.
- [13] S. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Pattern Anal. and Machine Intell.*, vol. 11, no. 7, pp. 674-693, 1989.
- [14] B. Tovar-Corona B and J. N. Torry, "Time-frequency representation of systolic murmurs using wavelets," *Comput in Cardiol*, 601-604, 1998.
- [15] D. Kumar, P. Carvalho, M. Antunes, J. Henriques, L. Eugénio, R. Schmidt, and J. Habetha, "Wavelet transform and simplicity based heart murmur segmentation", *Proc. of the Computers in Cardiology*, Valencia, Spain, September 2006.
- [16] N. Atanasov and T. Ning, "Isolation of systolic murmurs using wavelet transform and energy index," *Proc. Int. Congress on Image and Sig. Proc.*, pp.216-220, Hainan, China, May 27-30, 2008.
- [17] E. Delgado-Trejos, A. F. Quiceno-Manrique, et al. "Digital auscultation analysis for heart murmur detection." *Ann Biomed Eng.* 37(2):337-53, Feb. 2009.